Displacement, Velocity, Acceleration

Definition.

- **Displacement** (s): directed distance from a fixed origin.
- **Velocity** (v): rate of change of displacement with respect to time.
- **Acceleration** (a): rate of change of velocity with respect to time.

These are *vector* quantities — they have direction, carried by their sign. The corresponding *scalar* quantities are distance and speed.

Fact (Units) — displacement m; velocity $\mathrm{m\,s^{-1}}$; acceleration $\mathrm{m\,s^{-2}}$.
An acceleration of $3\,\mathrm{m\,s^{-2}}$ means the velocity increases by $3\,\mathrm{m\,s^{-1}}$ every second.

Example

A particle moves 8 m to the right, then 3 m to the left, taking 5 s in total. Find

1. the distance travelled and the final displacement;
2. the average speed and the average velocity.

Example

A car travelling at $90\,\mathrm{km\,h^{-1}}$ brakes uniformly to rest in 8 s. Find its acceleration in $\mathrm{m\,s^{-2}}$.

Textbook Exercises: SPS Course 4.9, Exercise 2 Q1–4

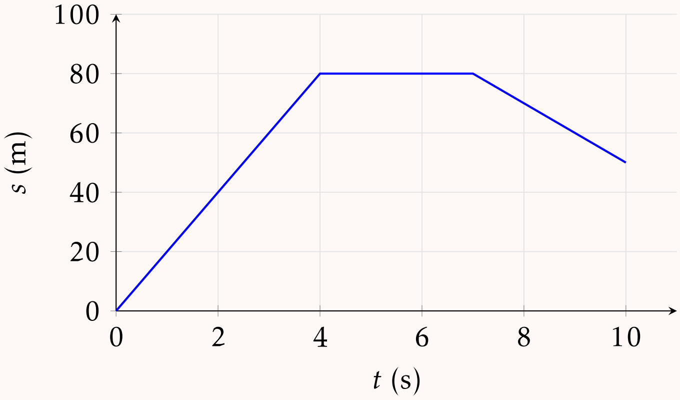
Displacement–Time Graphs

Fact — On a displacement–time graph:

- the gradient is the velocity (constant gradient = constant velocity);
- a horizontal section means the particle is stationary;
- negative gradient means the particle is moving back towards (or past) the origin.

Example

The graph shows the displacement of a particle (in m, s).



Find the velocity at $t = 3$, the velocity at $t = 9$, the average velocity over the 10 seconds, and the average speed.

Example

Fred and Harold travel separately from London to Brighton, 80 km away. Fred sets off at noon at 30 km h^{-1}; Harold sets off at 1 pm at 50 km h^{-1}. Find exactly when and where Harold overtakes Fred.

Textbook Exercises: SPS Course 4.9, Exercise 2 Q5, 6, 8, 11

Velocity–Time Graphs

Fact — On a velocity–time graph:

- the gradient is the acceleration;
- the area between the graph and the t-axis is the displacement (areas below the axis count as negative).

Example

A car accelerates uniformly from rest to $V\ \mathrm{m\,s^{-1}}$ in 8 s, holds that speed for 12 s, then decelerates uniformly to rest in 5 s. The total distance travelled is 370 m. Sketch the velocity–time graph and find V.

Example

A train accelerates uniformly from rest to $24\,\mathrm{m\,s^{-1}}$ in $60\,\mathrm{s}$, travels at this speed for 5 minutes, then decelerates uniformly to rest at $0.8\,\mathrm{m\,s^{-2}}$.

1. Sketch the velocity–time graph.
2. Find the total distance travelled.
3. Find the average speed for the whole journey.

Example

A ball is thrown vertically upwards at $20\,\mathrm{m\,s^{-1}}$ and is in flight for $4\,\mathrm{s}$ (take the acceleration to be $-10\,\mathrm{m\,s^{-2}}$ throughout). Sketch the velocity–time graph, and interpret the two triangles it makes with the t-axis.

Textbook Exercises: SPS Course 4.9, Exercise 2 Q7, 9, 10, 12 and Revision Exercise 4.9

Curved Graphs

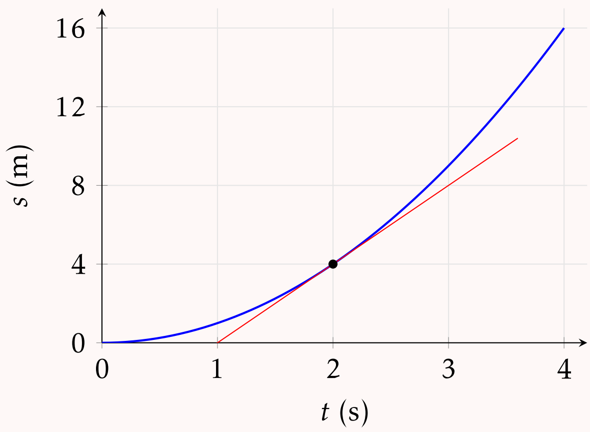
A curved displacement–time graph means the velocity is changing; a curved velocity–time graph means the acceleration is changing.

Fact — The gradient of a curve at a point is the gradient of the **tangent** at that point. Draw the tangent, form a right-angled triangle from it, and read off $\frac{\text{change in } y}{\text{change in } x}$ using the *axis scales* — not by counting squares.

Remark. The gradient is *not* the gradient of the line from the origin to the point.

Example

The graph shows $s = t^2$ for a falling particle, with the tangent drawn at $t = 2$. Find the velocity at $t = 2$.



Estimating the Area Under a Curve

Fact — Split the region into vertical strips and treat each strip as a trapezium:

$$\text{area of strip} \approx \frac{h}{2}(v_{\text{left}} + v_{\text{right}})$$

where h is the strip width. Whether this over- or under-estimates depends on which way the curve bends — decide from a sketch.

Example

The velocity of a particle is recorded each second:

t (s)	0	1	2	3	4
v ($\mathrm{m\,s^{-1}}$)	0	1.8	3.2	4.2	4.8

1. Estimate the distance travelled in the 4 seconds using four trapezia.
2. The velocity is in fact increasing with decreasing acceleration throughout. Is your estimate an over-estimate or an under-estimate?

Exercise. Sketch $y = x^3 - x$. Using tangents at a few points (and symmetry), sketch the graph of its *gradient* against x. What type of function does the gradient graph appear to be?

Textbook Exercises: SPS Course 4.9, Exercise 1 and Exercise 2 Q13–16, 18